Arc Resistance Coverage and Mho Expansion - The Devil is in the Details

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The topic of arc resistance coverage has been covered extensively throughout relaying history. However, through the recent years, there have been challenges to the way arc resistance has been calculated, and relays have become more complex which leads to misperceptions. This paper will cover: the history, development and concerns over arc resistance, relay response to fault conditions with realistic arc resistance coverage, trade-offs to obtain desired coverage, and practical recommendations to achieve desired coverage. Most importantly, it will address common misperceptions about mho expansion and what coverage an engineer may actually obtain from it.

Keywords—Mho expansion, arc resistance, positive-sequence memory voltage

I. INTRODUCTION

The phrasing "arc resistance coverage" was chosen over "fault impedance" to distinguish between a somewhat predicable value of arc resistance and an unpredictable resistive fault due to a foreign object.

Arc resistance coverage has been covered extensively throughout the years. It gained traction when addressed by Warrington in [1]. Warrington established the ill-effects that arc resistance had on distance relays, and attempted to provide clarity on actual arc resistance coverage. Since the first paper [1], other methods have been introduced [2,3,4], and these methods are compared extensively in [5]. Relay engineers typically use one of the methods listed previously, or the worst case result of any two of them. Then, depending on the utility practice, arc resistance coverage may be checked as a "nice to have", or it may actually influence a relay's overcurrent pickups and/or a distance element's reach setting.

Self-polarized mho distance elements have no expansive characteristics and are commonly referred to as faulted-phase polarized distance elements. Due to the fixed reach of self-polarized mho phase distance elements, they typically provide limited arc resistance coverage for phase faults (multi-phase faults). Resistive ground faults typically are detected by sensitive ground overcurrent elements, which are not limited by fixed reaches of mho ground distance elements.

There are expansion properties of mho distance elements depending on the polarization method and relay used. This means the actual characteristic expands beyond the static mho circle, thus providing increased resistive coverage. Older electromechanical (EM) relays that exhibited an expansion characteristic used cross- or quadrature-polarization (healthy-phase voltage) [6], and newer microprocessor relays typically use memorized positive-sequence voltage for polarizing. The resulting mho expansion may or may not provide the desired resistive fault coverage – depending on the relay and settings applied.

The goal of this paper is to assist a relay engineer in:

- Considerations when determining arc resistance
- How to best approach obtaining proper coverage
- How to best apply settings for resistive fault coverage using various mho relays of differing polarization

II. DEFINING FAULT RESISTANCE

A. Arc Resistance

It is first important to differentiate the fault impedance from arc resistance. Arc resistance is the result of a flashover which develops due to proximity of conductors to another phase or ground. This value is more predictable in that it can be estimated with margin when detecting faults. Many utilities calculate or apply fixed values when checking for resistive fault coverage. Phase faults should in most cases be evaluated based on calculated arc resistance. Ground faults may involve an arcing fault, or they can occur due to a foreign object with a "fixed" impedance. Reference [7] provides a thorough illustration for visualizing the various arcing resistance paths.

Arcing fault resistance is estimated through calculations, while a fixed resistance target due to a foreign object may come from past experience of the utility.

1) Phase Faults

Phase faults with arc resistance will typically occur between two phases. A balanced three-phase fault with arc impedance is rare. Phase-to-phase faults may start with a tree, wind or a balloon, but they most likely will develop into an arcing fault as the air ionizes, providing a lower impedance path. This simplifies predicting when resistive fault coverage is adequate for phase faults.

2) Ground Faults

Ground faults are more complex than phase faults. There are instances where insulators break down and an arcing fault begins; however, there are many cases where a foreign object comes in contact with a conductor. A tree is one of the more common cases, and the impedance is both high and unpredictable. The 100 ohm fault check came in reference to trees from Blackburn [4]. This value is applied as a fixed resistance check for ground faults quite frequently. This coverage check's success depends on the voltage level, and the source and line impedances. For instance, see Table I. It may not be realistic at lower voltage levels to detect a 100 ohm fault. The results assume an infinite source, no line impedance, and still can leave little margin for resistive fault coverage.

TABLE I. 100 OHM FAULTS

Voltage Level (kV)	Current (A)
345	1992
230	1328
115	664
69	398

B. Arc Resistance Calculation Method

Reference [5] analyzes the various methods of calculating fault impedance. For simplification purposes, those methods are evaluated in Table II for a select few fault values, and the naming is consistent with [5]: Warrington, Mason, Goda, Terzija and Koglin, and Blackburn, respectively. The 5th option is excluded due to the voltage level. A conductor spacing of 25 ft (7.62 m) is assumed. For the range of fault currents in Table II, Mason's (R_{A2}) method tends to be the most conservative.

TABLE II. COMPARISON OF ARC FAULT CALCULATION METHODS

Fault Current	R_{A1}	R_{A2}	R_{A3}	R_{A4}	R_{A6}
1,000A	13.80	13.75	7.28	6.55	11.0
10,000A	0.55	1.38	0.72	0.65	1.10
20,000A	0.21	0.69	0.36	0.33	0.55
30,000A	0.12	0.46	0.24	0.22	0.37
40,000A	0.08	0.34	0.18	0.16	0.28

C. Fault Location

The fault location does not necessarily determine if an arc will occur, but it does play a large factor in a mho distance element's ability to detect the fault. Close-in faults and end-of-the-line faults tend to be the most desirable places to check for resistive fault coverage, but these faults are located at the most limiting parts of the self-polarized mho circle for resistive coverage. The self-polarized mho has no expansion characteristic; its characteristic is the static circle. For instance, see fig. 1, where the resistive fault coverage is in red, and Z_R is the overreaching elements reach of the relay in question.

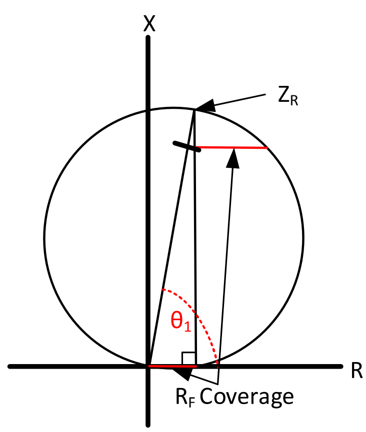

Fig. 1. Close-in and end-of-the-line coverage of a self-polarized mho

Since fig. 1 displays an overreaching element, there is some coverage at the remote bus, but less for a close-in fault. The maximum resistive coverage occurs for a fault at approximately 50% of the relay's reach. A fault at this location occurs at the center of the mho circle, meaning you have the full radius ($Z_R/2$) directly to the edge of your mho circle. Figure 2 shows the maximum coverage of the self-polarized mho element. If mho expansion is introduced, the coverage can change significantly, which will be covered in a later section.

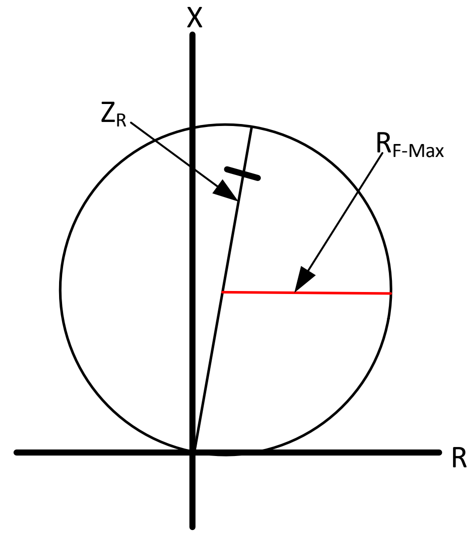

Fig. 2. Maximum self-polarized mho resistive coverage

To obtain greater arc resistance coverage, old practice with self-polarized mho distance elements was to change the line characteristic angle and extend the reach. This provided greater arc resistance coverage by tilting the mho circle towards the resistive axis, as shown in fig. 3. The reach itself was determined based on the difference in the line characteristic angle and the relay maximum torque angle, and the desired reach of the relay.

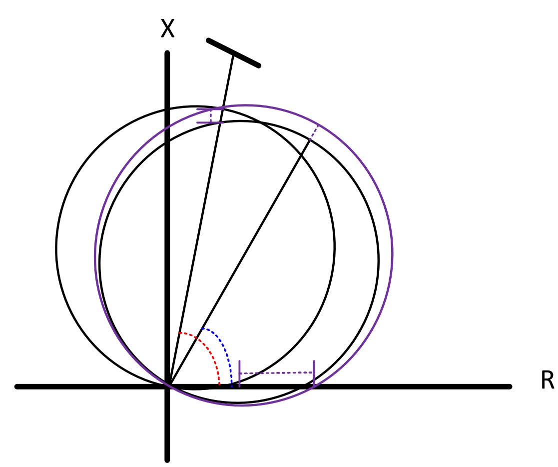

Fig. 3. Characteristic angle tilt effect

While the fault location does not affect the arc resistance, it is of concern to the relay engineer due to the effects it has on arc resistance coverage.

D. Fault Current

Arcing faults are a function of the fault current magnitude, thereby making them a function of the source feeding the fault. When two different line terminals feed a fault, there is an infeed effect and further reduction of the arc resistance because of the increased current flowing in the arc. The increased current tends to reduce the arc resistance; however, the infeed from the other terminal also increases the relays measured apparent impedance.

Resistive fault coverage is determined by each terminal's ability to clear the fault based on their source and various operating conditions. For example, fig. 4 shows a simple 230kV system where the resistive fault coverage can vary greatly depending on which terminal(s) feed the fault.

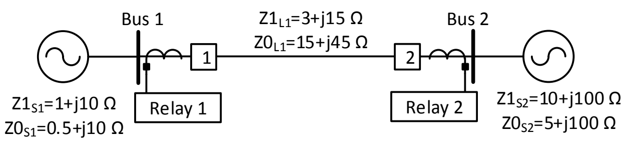

Fig. 4. Sample system

Using Warrington's method (1) of calculating the arc impedance, Table III shows the resulting arcing impedance and each relays apparent impedance measurement due to infeed. L is the arc length in meters and I is the current in the arc. The spacing between the conductors is 25ft (7.62 m) and the fault type is a phase-to-phase fault. Faults considered are line-end-open (LEO) – a fault at the end of the line with the breaker open – and close-in faults (CIF). From the results, Relay 2 with the weaker source has a more difficult time with the resistive fault. Also, depending on the location of the fault, the apparent impedance measured varies.

$$R_{A1} = \frac{28,707.35\frac{V}{m}}{I^{1.4}} \tag{1}$$

TABLE III. APPARENT IMPEDANCE DUE TO ARC RESISTANCE

Fault	RF (Ω)	Relay 1 (Ω)	Relay 2 (Ω)
CIF Bus 1	0.41	0.22∠0°	15.97∠70°
LEO fault at Bus 1	13.92	NA	18.00∠56°
CIF Bus 1 with BKR 2 open	0.46	0.23∠0°	NA
CIF Bus 2	1.21	15.45∠76°	3.00∠3°
CIF Bus 2 with BKR 1 open	11.43	NA	5.72∠0°
LEO fault at Bus 2	1.66	15.48∠76°	NA
50% fault	0.81	7.75∠75°	8.76∠60°
50% fault with BKR 1 open	12.65	NA	10.84∠44°
50% fault with BKR 2 open	1.00	7.76∠75°	NA

III. THE RELAY'S RESPONSE

Both EM relays and newer microprocessor mho distance elements are based on the phase-comparator generated mho circle principles. These phase comparators typically test the angle between signals (2) and (3), which results in the mho characteristic of a circle with a boundary where the angle difference between S_1 and S_2 is 90°. Signal S_1 is the difference between the operate (IZ_R-V) and the restraint (V) quantities. S_2 is the polarizing signal which can take different forms. EM relays compare the signals (4) based on torque, and (5) is the equivalent phase-comparator in microprocessor relays [8].

$$S_1 = I \cdot Z_R - V \tag{2}$$

$$S_2 = V_P \tag{3}$$

$$S_1 \cdot S_2 = 0 \tag{4}$$

$$\mathrm{Re}\left[S_1 \cdot S_2^*\right] = 0 \tag{5}$$

V_P – polarizing voltage
I – measured current
Z_R – relay setting
V – measured voltage at the relay

A mho elements response to a fault depends on the polarization method used. Most modern relay manufacturers have settled on using positive-sequence memory voltage (V1MEM) for the polarizing quantity (S_2). Unfortunately for the relay engineer, each manufacturer holds V1MEM for different durations of time. This has a significant effect on whether the resistive fault coverage provided by mho expansion can realistically be counted on for the expected fault clearing.

A. Polarization

Memory voltage is the pre-fault voltage that is held by the relay for polarizing reference (S_2). This dates back to electromechanical (EM) relaying days when a simple LC tuning circuit would hold the pre-fault polarizing voltage for a couple of cycles, which is shown in fig. 5 [9]. The larger amplitude waveform is a reference. The smaller waveform is the current flowing in the relays polarizing circuit.

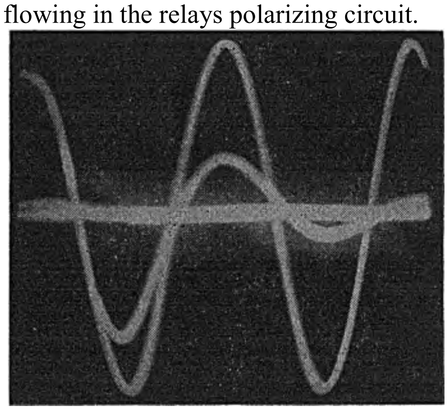

Fig. 5. EM memory trace from [9]

The intention of the LC circuit was to maintain the relays polarizing signal long enough for the relay to operate for a close-in fault with zero-voltage. Some EM relays also used cross-polarization or healthy-phase(s) voltage with a phase shifting circuit to provide the polarizing signal for non-three-phase faults.

It was discovered in 1965 by Wedepohl that using a voltage other than the faulted-phase voltage for polarizing caused the mho relay characteristic to expand [6]. This remarkable discovery (expansion) revealed the benefit of additional arc resistance coverage (as compared to the static circle).

Figure 6 shows a popular ground distance EM relay's polarizing, restraint and operate AC connections. From the figure, we can see that the restraint (RES) and operate (OP) connections of A-phase align with the polarization (POL) connections phases B and C. For a ground fault on A-phase, this provides B-C-phase polarization (healthy-phase polarizing), allowing the relay to operate for close-in zero-voltage A-phase faults and also provide the bonus of additional resistive fault coverage due to the expansion.

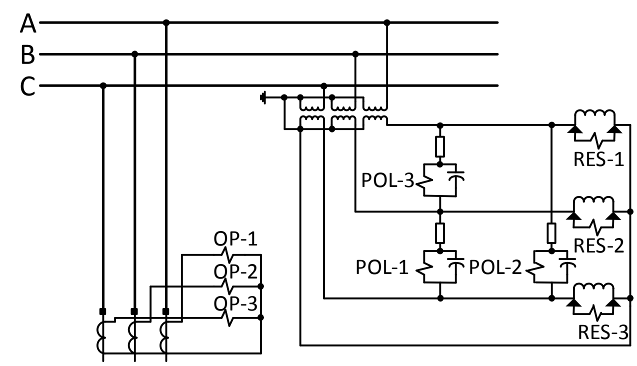

Fig. 6. EM AC connections

If one were to assume a bolted single-line-to-ground fault at the relays terminals in fig. 6, with a Z0/Z1 ratio of one and no fault impedance, the polarizing voltage measured at the relay would be the full phase-to-phase voltage (1.732 times the line-to-neutral voltage). This voltage is shifted by a tuned circuit to bring the polarizing voltage in-phase with the pre-fault faulted-phase voltage (e.g. the B-C voltage is shifted 90° to bring it in phase with A-phase pre-fault voltage).

While defining the polarizing source is simple, the analysis of the element in operation and system interaction is much more complex. For instance, fig. 7 shows example phasors of a fault that has some fault resistance and is within the elements zone of protection, while V_P is the memorized pre-fault voltage. There is fault resistance because V, I, and IZ_R-V are not in phase with V_P. The fault is within the zone of protection because the angle between V_P and IZ_R-V does not exceed 90°.

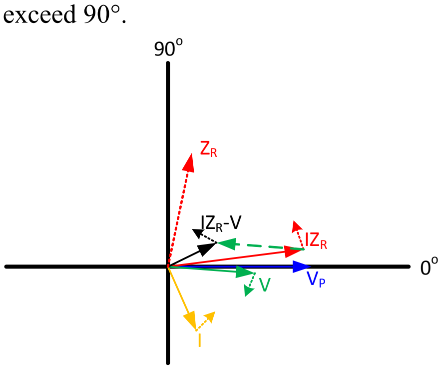

Fig. 7. Distance relaying voltage and current phasors

As fault resistance is added in fig. 7, I becomes less lagging which causes IZ_R to become more leading. Also, as the fault resistance is added, V begins to lag the initial memorized voltage, V_P. Since angle comparators test the angle between V_P and IZ_R-V, one can easily conclude that IZ_R-V moves away from the polarizing signal with each incremental increase in fault resistance. In the case where V_P is fixed at the pre-fault voltage, IZ_R-V must move 90° from the fixed angle. However, if this was a self-polarized mho, V_P is V and would begin to pull away from the signal IZ_R-V, making the two signals diverge from each other as resistance is added.

The above is further verified in Table IV, where the angles are displayed for the system in fig. 4. A three-phase fault has occurred at 50% of the line and the Z_{BC} mho loop is being evaluated. Because the Z_{BC} loop is being evaluated, everything is shifted by negative 90° from fig. 7. The results show V1MEM fixed at negative 90° (the normal position of VBC prior to the fault) and lagging IZ_R-V more with each increase in R_F, until R_F exceeds 11Ω, where the angle difference has become greater than 90° (for RF = 11Ω the angle difference is 89.4°). We can do the same angle difference evaluation when compared to V, which exceeds 90° after R_F exceeds 7Ω (this would be the self-polarized approach).

TABLE IV. QUANTITY ANGLES

R_F (Ω)	IBC·Z_R-V	V1MEM	VBC	IBC·Z_R
1	-82.5°	-90°	-97.1°	-90°
3	-61.2°	-90°	-103.5°	-83.9°
5	-41.8°	-90°	-107.7°	-78.1°
7	-25.3°	-90°	-110.1°	-72.8°
9	-11.7°	-90°	-111.2°	-68.0°
11	-0.6°	-90°	-111.4°	-63.7°
13	8.4°	-90°	-111.2°	-59.8°

Another way of visualizing the results from Table IV is fig. 8. The operate signal (IBC·Z_R-V) is compared to the two possible polarizing signals, VBC and V1MEM. When the difference between the two signals exceeds 90°, the distance element ceases to operate. Again, for the self-polarized mho, this happens around 7Ω, while the positive-sequence voltage polarized mho happens around 11Ω.

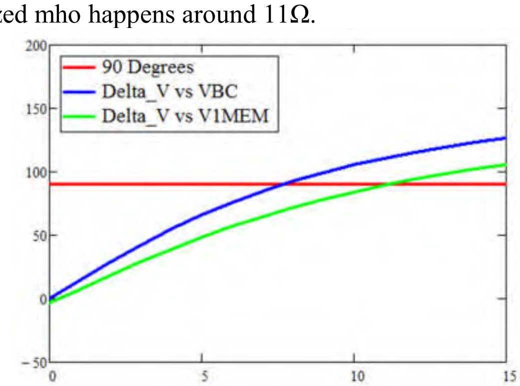

Fig. 8. Operate signal vs different polarizing signals

Further, it is convenient to view the voltage and current phasors from fig. 7 on an impedance diagram, such as fig. 9. Figure 9 is the result of dividing the voltage signals by I for convenience. The resulting Z_P (V1MEM) and Z (V) maintain the same phase angle relationships to Z_R-Z (IZ_R-V). We can do this because we are comparing the phase angle of two complex values. Simply diving each by the same value (I) does not change their phase position relative to each other.

Z_P – resulting impedance from the polarizing voltage
Z – measured (apparent) impedance at the relay
Z_L – line impedance
Z_R – reach of relay

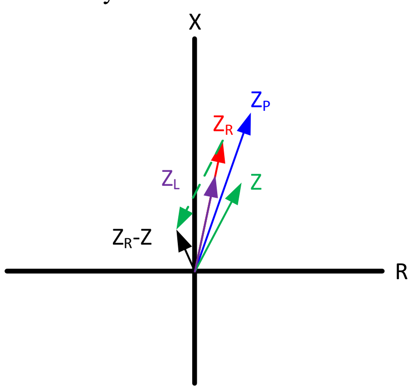

Fig. 9. Distance relaying equivalent impedance phasors

Now, consider a mho phase distance relay which is polarized from the un-faulted phase. Full line-to-neutral voltage for a phase-to-phase fault is measured as the polarizing quantity, and the angle is compensated for by a tuned circuit – bringing it in phase with the faulted phase-to-phase voltages pre-fault value. Since there is no angle difference between this signal and the pre-fault positive-sequence voltage used before, the operating characteristic remains the same as previously seen in Table IV.

This expansion functionality was later translated into the microprocessor relays in the 1990s, except that instead of using cross-polarization, positive-sequence voltage was used [8,10]. Equation (6) is the result of inserting (2) and (3) into (5), when V_P is the measured voltage V, while (7) is the result when V_P is V1MEM. Both of these equations are equal to zero at the balance point of the mho distance element.

$$\mathrm{Re}\left[\left(I \cdot Z_R - V\right) \cdot V^*\right] = 0 \qquad (6)$$

$$\mathrm{Re}\left[\left(I \times Z_R - V\right) \times V1MEM^*\right] = 0 \qquad (7)$$

** is the complex conjugate*

Table V shows the resulting values from the Phase Angle Comparators in (6) and (7). At approximately 90°, a cosine comparator will produce a negative value, resulting in no trip. Therefore, any angle less than 90° results in a trip. Notice that the 90° difference is exceeded at the same RF values in Table IV, showing agreement with the previous results.

TABLE V. PHASE ANGLE COMPARATOR'S ANGLES IN DEGREES

RF	Equation (6)	Equation (7)
1	14.6	7.5
3	42.3	28.8
5	65.9	48.2
7	84.8	64.7
9	99.5	78.3
11	110.7	89.4
13	119.5	98.4

The method of using V1MEM has been mostly adopted by U.S. relay manufacturers. Each relay manufacturer handles memory voltage differently, and it may differ between models from the same manufacturer. Three relays are examined in this paper, which are described below:

1. Relay Manufacturer 1 (RM1) has a programmable memory duration, where the full pre-fault voltage is held for a user specified amount of time. This allows the user to extend the duration in which expansion can be expected.
2. Relay Manufacturer 2 (RM2) uses an algorithm to hold the memory voltage, but also track the system voltage, which results in a decaying memory voltage over time (decays to the actual system voltage). There are two options, short or medium length time constants.
3. Relay Manufacturer 3 (RM3) uses a definite time in which the full memory voltage is held. There is no option to track or hold the voltage.

The relay engineer must understand their system and decide how closely the relay must track the voltage and frequency [10,11]. In other words, high-inertia systems will most likely remain fairly constant in terms of voltage and frequency, while weak systems (low-inertia) may vary significantly. If the system and memorized voltage begin to drift apart, voltage based mho elements may yield incorrect results and result in misoperation.

B. Mho Expansion

The phenomenon of mho expansion dates back to its discovery as recorded in Wedepohl's paper [6], and more recent references include [10,12,13]. The relays reviewed in this paper all use memorized positive-sequence voltage for polarization, but the methods in which it is applied differs. The reader should understand that the voltage magnitude of the polarizing signal does not affect expansion; it is the angle difference as the system changes that matters, as shown in fig. 7.

1) Relay Manufacturer 1

The duration of mho expansion for RM1 consists of a fixed, settable time. This relay holds the memory voltage for the settable amount of time.

2) Relay Manfacturer 2

RM2 has a variable expansion characteristic that shrinks with time, depending on the setting of short or medium length memory voltage time constant and the measured voltage. Figure 10 shows results of a test with the two different setting options, where rated secondary voltage is applied to the relay, then removed simulating a close-in three-phase fault. For the shorter time constant, the memory voltage quickly decays along with the measured positive-sequence voltage. The polarizing voltage decays so rapidly that an instantaneous mho element may not respond, depending on the arc resistance and other system conditions. On the other hand, we see that with the longer time constant the mho elements expansion decays less quickly, as expected. As such, we can be confident with the medium time constant approach for some instantaneous arc resistance coverage, but a margin should be applied. For faults that produce very low measured positive-sequence voltages, the relay will switch to a longer time constant automatically, which is seen in fig. 10 by the near flat line of both traces after the measured positive-sequence voltage reaches a predefined value.

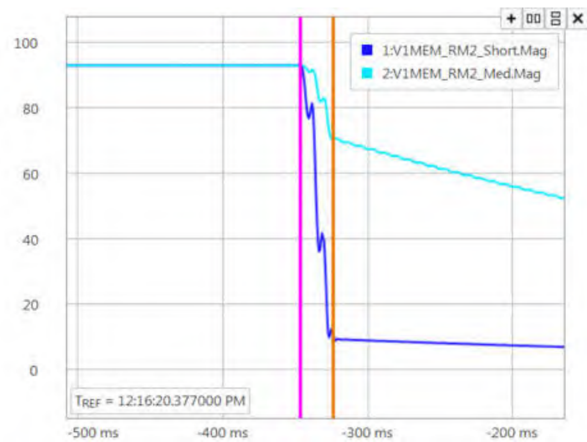

Fig. 10. Relay Manufacturer 2 (RM2) time constant decay

3) Relay Manufacturer 3

RM3 has a definite time duration, where if the fault condition still exists after that time, the time delayed mho element may drop out due to the reduction of the expansive characteristic.

C. Fault Type Effect on Positive-SequencePolarizing

Expansion is the same regardless of fault type when memory voltage is at the full pre-fault voltage; otherwise, it varies based on fault type.

a) Three-Phase

Three-phase faults experience full expansion at fault inception, only to shrink back to self-polarized form once memory voltage expires.

b) Phase-to-Phase

For phase-to-phase faults, the corresponding mho elements do not shrink all the way back to their self-polarized characteristic. This has to do with using positive-sequence voltage and the way the sequence networks are interconnected [13,14]. One can expect half the source impedance to be "measured" once memory voltage has expired, which results in half the expansion. This is referred to in this paper as "fixed" expansion, which can be seen in fig. 11 and simply means the polarizing quantity is the actual measured positive-sequence voltage (not memory).

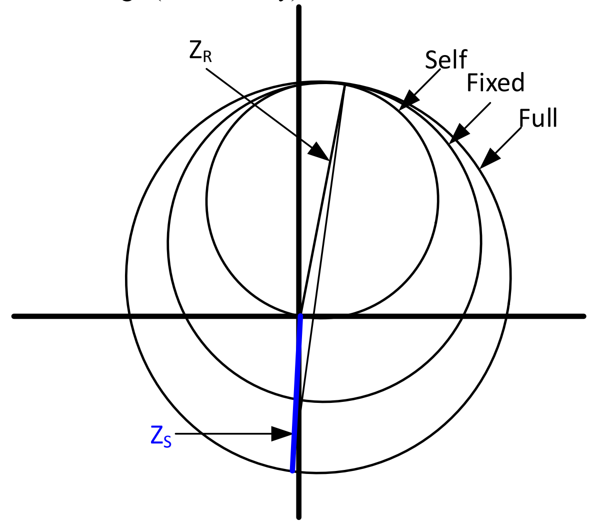

Fig. 11. Self, fixed and full expansion characteristics

c) Single-Line-to-Ground

Single-line-to-ground faults can have arcing resistance or fixed impedance faults. As such, estimating expansion is much more complicated for the ground mho element. The fixed expansion will be a function of zero-sequence and positive-sequence values, of the source and line impedances (8). If one was to assume that the source impedances are close in value, and that the zero-sequence to positive-sequence line impedance ratio is a typical ratio of three, the fixed expansion is around 40% of the original positive-sequence source impedance. Again, fortunately, expansion of mho elements for ground faults is not as critical because we have sensitive ground overcurrent elements available.

$$Z_{S_{Fixed}} = \left[\frac{1 + \frac{Z0_S}{Z1_S}}{2 + \frac{Z0_L}{Z1_L}} \right] \quad (8)$$

Since the double-line-to-ground fault is the combination of phase-to-phase and line-to-ground faults, it will not be analyzed. Depending on whether the fault impedance lies between the two phases or ground, R_F can drive the fault currents to more closely resemble either fault.

D. Fault Location

Mho expansion benefits are highly dependent on fault location. When evaluating a self-polarized mho element (which has no expansion), the remote bus fault obviously results in greater arc resistance coverage for the overreaching element because we are not at the end of the zone. However, a close-in fault benefits the most from expansion. Figure 12 illustrates the change in arc resistance coverage due to expansion depending on the fault location and source.

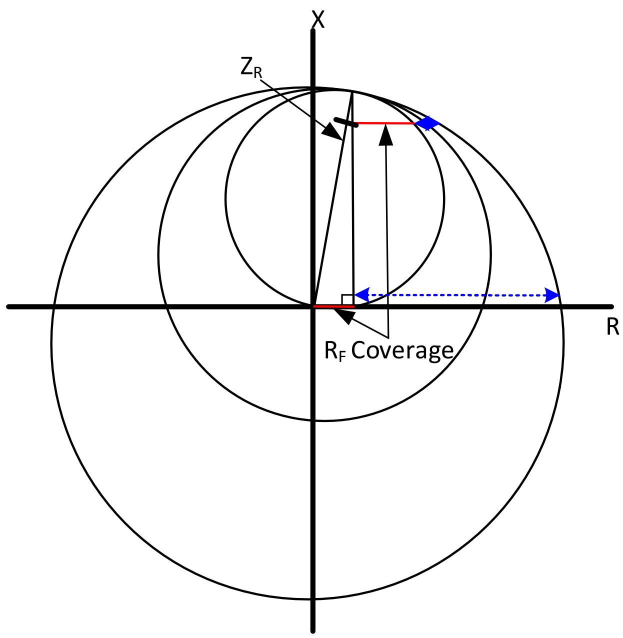

Fig. 12. R_F coverage at the beginning and end of zone

IV. Setting a Relay for Resistive Fault Coverage

In general, quadrilateral or offset self-polarized mho distance elements offer fixed arc resistance coverage for phase (multi-phase) faults. For ground faults, ground overcurrent elements can be relied upon for arc resistance coverage because of their sensitivity.

The following subsections will focus on mho elements due to their variable arc resistance coverage, and will use the example system in fig. 4. Zone 1 and 2 phase distance elements will be assumed at typical 80 and 120 percent margins, respectively.

A. Calculating Arcing Resistance

Of the many different methods compared in [5], Mason's (9) from [2] is the most conservative when given a wide range of arc resistance values and will be used in this example. L is the arc length in meters and I is the current in the arc.

$$R_{A2} = \frac{1804.46 \frac{V}{m}}{I} \cdot L \qquad (9)$$

The resulting arc resistance values are shown in Table VI, for three-phase (3PH) and phase-to-phase (PP) faults. These values are in columns 1 and 3. Also shown is the resulting apparent impedance for Relay 1, in columns 2 and 4.

TABLE VI. Relay 1 Apparent Impedance Due to Arc Resistance

Fault	**RF_{3PH} (Ω)**	**Relay 1 (Ω)**	**RF_{PP} (Ω)**	**Relay 1 (Ω)**
CIF Bus 1	0.96	1.04∠0°	1.11	0.60∠0°
CIF Bus 1 with BKR 2 open	1.04	1.04∠0°	1.20	0.60∠0°
CIF Bus 2	2.10	15.99∠69°	2.42	15.65∠73°
LEO fault at Bus 2	2.62	16.01∠69°	3.03	15.66∠73°

The resulting arc resistance values pertaining to Relay 2 are shown in Table VII, columns 1 and 3. Also shown are the corresponding resulting apparent impedances for Relay 2 in columns 2 and 4.

TABLE VII. Relay 2 Apparent Impedance Due to Arc Resistance

Fault	**RF_{3PH} (Ω)**	**Relay 2 (Ω)**	**RF_{PP} (Ω)**	**Relay 2 (Ω)**
CIF Bus 1	0.96	21.12∠45 °	1.11	17.93∠56°
LEO fault at Bus 1	11.99	21.21∠45°	13.84	17.98∠57°
CIF Bus 2	2.10	10.43∠3°	2.42	6.01∠3°
CIF Bus 2 with BKR 1 open	10.41	10.41∠0°	12.02	6.01∠0°

Calculating the arc resistance coverage when including mho expansion (for plotting on the R-X diagram) can be done with a known source impedance value, using (10). A detailed explanation of (10) can be found in the Appendix. Equations (11), (12) and (13) make up the variables in (10), where "n" is the distance to the fault. Z_S (source impedance) can dropout, or be changed to a fixed value as needed.

$$R_F = \sqrt{|Radius|^2 - Offset_{Imag}^{\,2}} - Offset_{Real} \qquad (10)$$

Where,

$$Radius = \left| \frac{Z_R + Z_S}{2} \right| \qquad (11)$$

$$Offset_{Imag} = Im(n \cdot Z_L) - \frac{Re(Z_R - Z_S)}{2} \qquad (12)$$

$$Offset_{Real} = Re(n \cdot Z_L) - \frac{Re(Z_R - Z_S)}{2} \qquad (13)$$

B. Fault Location

The location of the fault is a preferential consideration. The fault locations chosen for this example are close-in and remote line-end-open faults, where arc resistance coverage is the most limited for the static characteristic. Referring to Tables VI and VII, 3PH faults fare worse than phase-to-phase faults at the beginning and edge of the zone of protection. Also, in this example, faults with infeed result in similar apparent impedance values when compared to the radial case where the remote source is removed. There is an interesting takeaway from this example, as the infeed is removed, the apparent impedance does not change much. This will most likely be the case when there is not significant transfer impedance between the two source equivalents. Practically, this cannot be assumed and must be checked each time.

C. Fault Types

Fault types do not determine how much the memory polarized mho elements expand initially, but they do influence the extent to which one can rely on expansion <u>after</u> memory voltage expires/decays. When the memory is gone and only the measured positive-sequence is available, we are at the “fixed” expansion as shown in fig 11. This is simply because the positive-sequence fault voltage depends on the fault type.

1) Three-Phase Faults

For the results in Tables VI and VII – once memory voltage expires – the self-polarized mho circle should be verified to cover any desired arcing fault since there is no fixed expansion with this fault type. After memory is gone it just exhibits a self-polarized characteristic.

For a close-in fault, Relay 1’s arc resistance coverage will need to be 1.04Ω plus margin. Using (10) and recognizing that the source impedances drop out of the equation, the self-polarized characteristic covers up to 2.4Ω of arc resistance. The full range of Relay 1’s coverage is found in Table VIII. Zone 1 benefits the most from the expansion, as shown in fig.12.

TABLE VIII. RELAY 1 RF COVERAGE ANALYSIS

Coverage Consideration	RF Z1 (Ω)	RF Z2 (Ω)
3PH Self-Polarized	2.4	5.7
3PH Full Expansion	11.8	7.3
PH-PH Self-Polarized	2.4	5.5
PH-PH Fixed Expansion	8.8	6.6
PH-PH Full Expansion	11.8	7.3

On the other hand, Relay 2 has the same self-polarized arc resistance coverage (assuming the reach values are the same), so the arc resistance of 11.99Ω exceeds the self-polarized arc resistance coverage. If one desires instantaneous tripping fault coverage using a mho element, expansion would have to be relied upon. The full range of Relay 2’s coverage is found in Table IX.

TABLE IX. RELAY 2 RF COVERAGE ANALYSIS

Coverage Consideration	RF Z1 (Ω)	RF Z2 (Ω)
3PH Self-Polarized	2.4	5.7
3PH Full Expansion	31.4	13.6
PH-PH Self-Polarized	2.4	5.7
PH-PH Fixed Expansion	23.5	10.9
PH-PH Full Expansion	31.4	13.6

For a fault at the edge of our zone, the desired arc resistance coverage of Relay 1 is 2.62Ω. Using (10), the calculated arc resistance coverage at the edge of zone for a self-polarized mho is 5.7Ω. This leaves some margin resulting in adequate arc resistance coverage. Relay 2 does not have adequate arc resistance coverage for the 10.41 ohm fault at the edge of the zone without relying on expansion.

2) Phase-to-Phase Faults

For this example, the self-polarized mho provides sufficient coverage for Relay 1, while Relay 2 may or may not have sufficient coverage depending on the scenario. Fixed expansion can be relied upon for phase-to-phase faults, if needed.

In general, if the three-phase self-polarized arc resistance coverage is satisfied, the phase-to-phase coverage is satisfied. This is because a phase-to-phase fault results in both the faulted phases sharing the arc impedance, which reduces the arc resistance effect by a factor of two [15].

3) Single-Line-to-Ground Faults

Single-line-to-ground faults are not analyzed herein due to the availability of sensitive ground overcurrent elements to provide that coverage as needed. Introducing fixed values of fault resistance due to a foreign object or tower strapping is highly subjective, and should be performed based on experiences with the specific system.

If one desires instantaneous mho coverage, the same analysis as the previous subsection can be performed. However, the ground fault coefficients will need to be substituted into (10) for the appropriate fixed expansion coverage.

4) Expansion Analysis Due to Fault Type

The previous results for Relays 1 and 2 vary quite drastically. Relay 1 does not need expansion to reliably detect arcing faults, while Relay 2 needs expansion for three-phase faults and some phase-to-phase faults. If three-phase arc resistance coverage is not of concern due to their unlikelihood, only phase-to-phase faults may be considered – which benefit from fixed expansion.

D. Other Considerations

Load flow and non-homogeneity can cause results to vary when using (10) [16]. These conditions should be accounted for by adding margin into arc resistance coverage analysis. Load flow will also affect the various fault types differently. Depending on the relay in question, load encroachment can limit the arcing resistance coverage [17].

There are specific situations where quad elements may be useful, but for the most part, they are not needed for sufficient arc resistance coverage [17]. One caution in applying quad elements is that overreaching elements should typically be avoided when the surrounding system incorporates mho distance elements. Overreaching quad elements maintain their arc resistance coverage to the edge of their zone, while mho distance elements have the most limited arc resistance fault coverage at the beginning of their zone. This can result in gaps in coverage and miscoordination between the two, as seen in fig. 13.

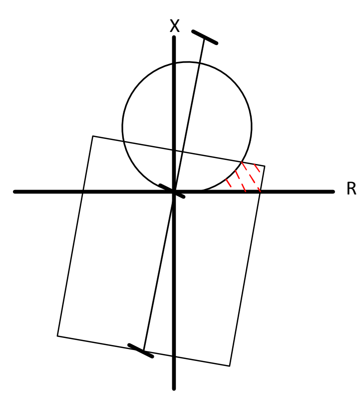

Fig. 13. Mho and quad coordination

E. Relay's Time Constant

The relays time constant and/or algorithm that holds memory voltage for polarizing is crucial when analyzing mho expansion. In general, it is more conservative to only consider self-polarized or fixed expansion (after expiration of memory voltage) when evaluating the coverage of these elements. If it is important that expansion cover specific faults, it is preferred that the distance elements be set to operate instantaneously, rather than risk corruption of the memorized voltage over time. Time-delayed elements may require too much time between fault inception to operation of the breaker to depend on memorized positive-sequence voltage, depending on the system and conditions at hand. Settable time constants should have at least two cycles of memory voltage to allow the relay to process instantaneous protective elements. Other system conditions may require a longer time constant.

Figures 14 and 15 show the various relay manufacturers performance when evaluating close-in and line-end-open faults, for Relay 2 in Table VII. The line charts at the bottom of each figure represent the phase distance elements duration in which it detects the fault. All relays have the capability to reliably detect the fault for at least a short duration. The line-end expansion benefits for RM2 are short-lived.

V. Conclusion

After comparing the different methods of retaining memorized positive-sequence voltage, mho expansion should only be relied upon after careful analysis when ensuring adequate arc resistance coverage. The following should be considered when attempting to calculate arc resistance fault coverage.

- Time-delayed elements should typically only rely on self-polarization or fixed expansion for arc resistance coverage.
- When considering mho expansion for arc resistance coverage, a time constant should provide a minimum of two cycles of pre-fault voltage for relay processing time. Short time constants should typically not be used when considering arc resistance coverage.
- Instantaneous tripping mho elements may provide adequate arcing resistance coverage provided by memory expansion when the appropriate time constant is used.
- Fixed expansion can be calculated for phase-to-phase faults using (10), using half the source impedance for mho elements that trip with a time delay.
- Instantaneous tripping mho elements can provide expanded resistive fault coverage for a varying amount of time, depending on the relay in question and time constant chosen.
- Memorized voltage is subject to corruption in low-inertia (weak) systems due to the change in frequency. The measured fault voltage and current phasors rotate with respect to the memorized pre-fault voltage as frequency deviates.
- Cross-polarized mho elements are not subject to corruption of memorized voltage due to frequency deviation because the un-faulted phase voltage(s) track with the system frequency. However, three-phase zero-voltage faults would still require some type of memory polarization for correct operation.
- Use a consistent method of calculating arc resistance. There are more accurate methods as of today [3], but if adequate margin is applied, any of the methods previously referenced will provide a reasonable estimate.
- Sensitive negative- and zero- sequence overcurrent elements provide superior resistive fault coverage for ground faults if applied.
- Quadrilateral distance and directionally-secured self-polarized offset-mho elements provide predictable resistive fault coverage, and do not require balancing system voltage and frequency tracking with resistive fault coverage. Care should be used when mixing quadrilateral in a system with mho elements.
- It is important to understand how computer modeling and simulation tools handle memory polarizing to verify their results.

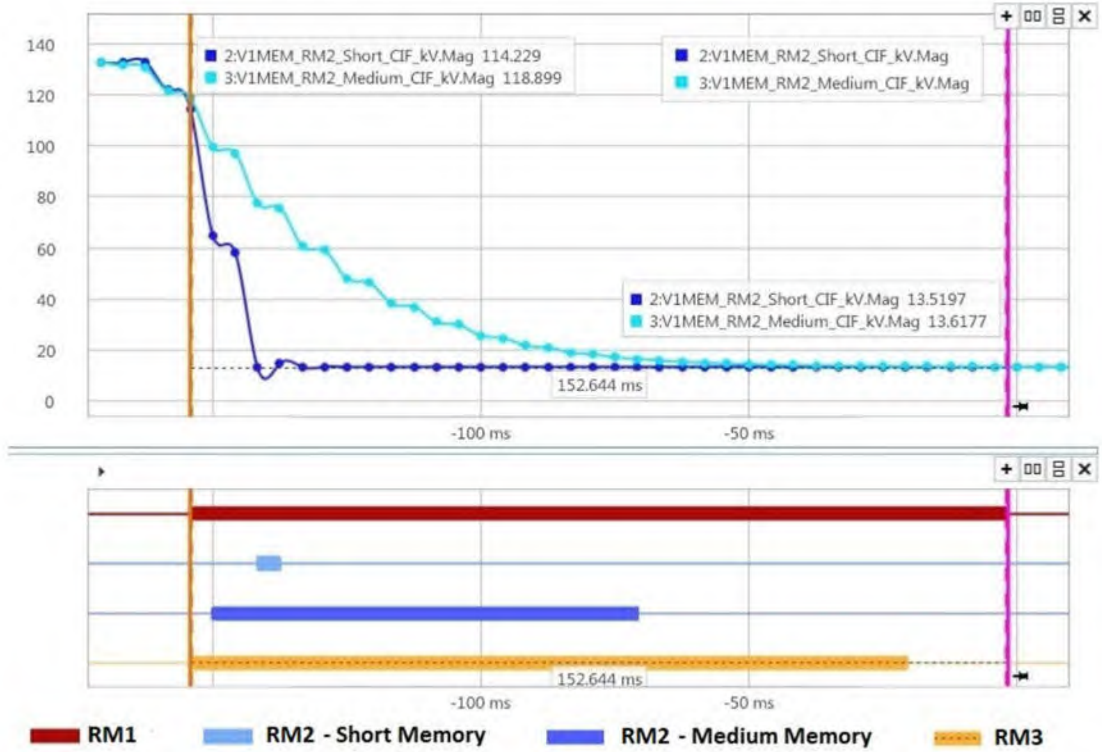

Fig. 14. Relay 2 close-in fault with relay manufactuers 1, 2 and 3

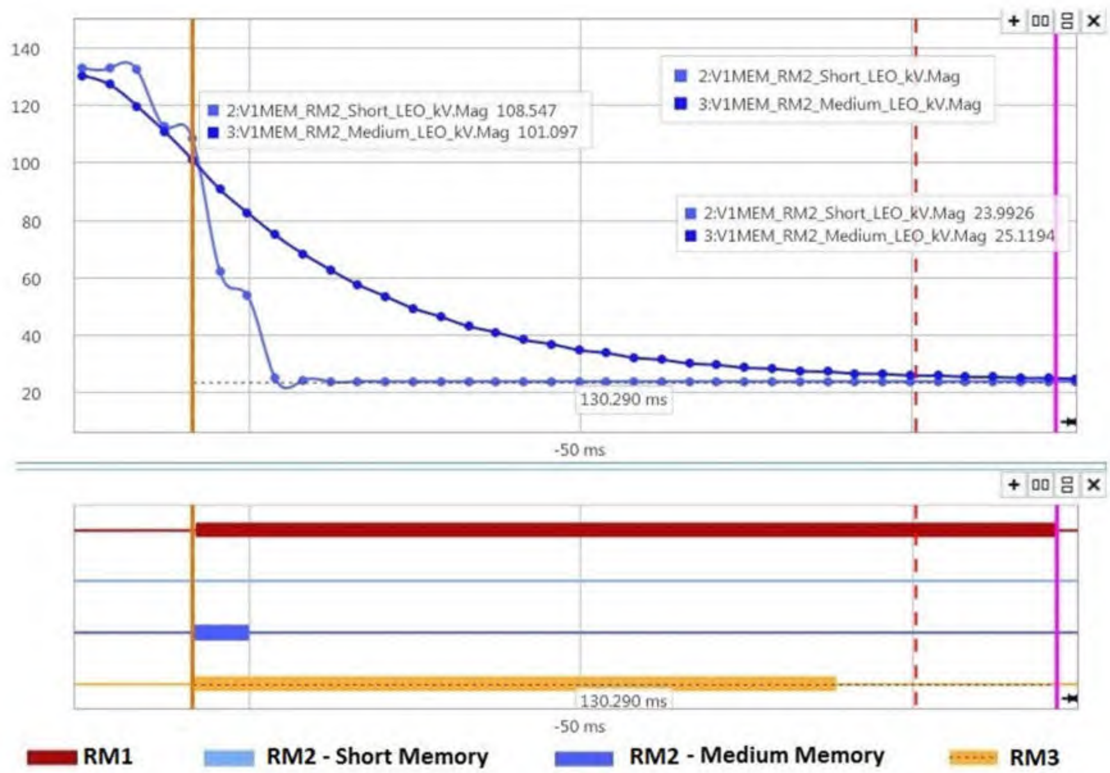

Fig. 15. Relay 2 line-end open fault with relay manufacturers 1, 2 and 3

VI. Appendix

A. Calculating Arc Resistance Coverage

Predicting/modeling resistive fault coverage due to mho expansion is very difficult. There are two practical approaches that can be taken, calculate the maximum coverage at fault inception, or calculate the minimum expansion after decay of the memorized positive-sequence voltage. At present, it is the authors' opinion that full expansion should not be considered in arc resistance coverage, unless a fixed time-constant setting is available, or a healthy margin is applied for instantaneous tripping mho elements only.

In order to determine how much arc resistance fault coverage the mho element is capable of, one must first start with system impedances. Since expansion is based on the source impedance, the line and the source impedances develop the circle. To start, the self-polarized mho circle is centered at half the line impedance (14). Its radius is half the magnitude of the line impedance (15).

$$C_{Self} = \frac{Z_R}{2} \tag{14}$$

$$R_{Self} = \left| \frac{Z_R}{2} \right| \tag{15}$$

To include the source impedance for expansion, equations (14) and (15) become (16) and (17).

$$C_{V1MEM} = \frac{Z_R - Z_S}{2} \tag{16}$$

$$R_{V1MEM} = \left| \frac{Z_R + Z_S}{2} \right| \tag{17}$$

The resulting expansion effects can be seen in fig. 16. The center of the circle due to expansion is directly related to the ratio of the source impedance to the line impedance. In other words, no source impedance (infinite bus) means no expansion.

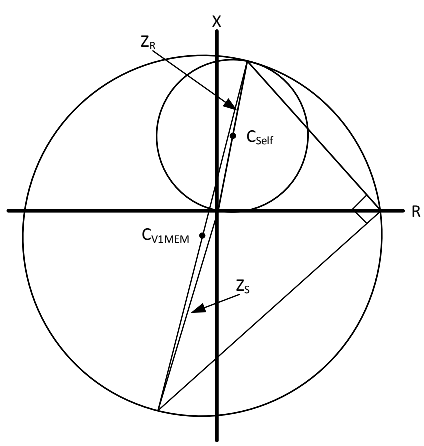

Fig. 16. Self and full expansion

Now that the expansion can be visualized, the resistive fault coverage can be broken up into real and imaginary parts, and using Thale's theorem, the resistance can be solved for. Observing fig. 17, we see that we can form a triangle out of the circle to calculate the resistance.

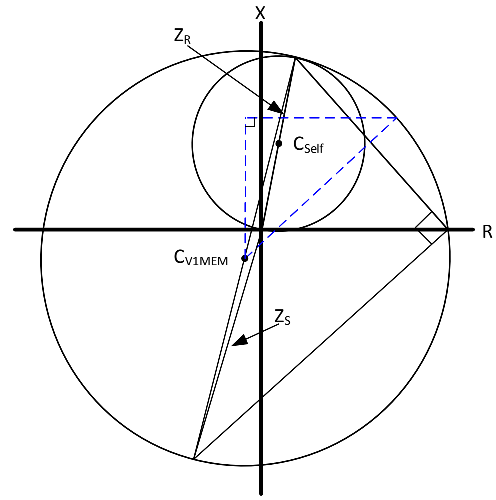

Fig. 17. Expansion broken out into triangle

Using Thale's theorem, we know that any line that passes through the center of a circle from end-to-end, creates a 90 degree angle with any other point on the circle. Using the center and known line impedance, the lengths of the triangle can be constructed as in fig. 18.

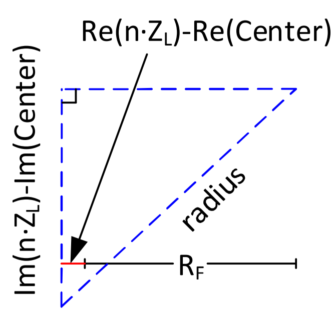

Fig. 18. Expansion triangle

Using Pythagorean's theorem (18), the individual lengths and circle components form (19), which was previously shown in a reduced form (10).

$$a^2 + b^2 = c^2 \tag{18}$$

$$R_F = \sqrt{\left|\frac{Z_R + Z_S}{2}\right|^2 - \left[\mathrm{Im}\left(n \cdot Z_R\right) - \mathrm{Im}\left(\frac{Z_R - Z_S}{2}\right)\right]^2 - \left[\mathrm{Re}\left(n \cdot Z_R\right) - \mathrm{Re}\left(\frac{Z_R - Z_S}{2}\right)\right]} \tag{19}$$

Evaluating self-polarization, Z_S drops out, and we are left with half the line impedance, and the differences between the center and the fault location. Figure 19 shows the resulting triangle for a fault past half of the reach.

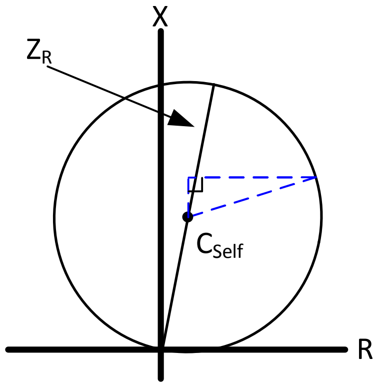

Fig. 19. Self-polarization analysis

B. *Mho Expansion*

Mho expansion is based on the premise of two comparators [6]. Self-polarized mho elements use the measured voltage as the polarizing voltage, and as a result, zero-voltage faults produce zero torque. Reference [6] starts with the self-polarized analysis and then analyzes three different methods of polarization, all expanding back to the source, but scaled by some constant. Unfortunately, none of the options analyzed were that of typical EM relay connections in the U.S., such as shown in fig. 6. Typical comparator equations (2) and (3) can be advantageously translated into impedances. This is done by doing simple circuit analysis for a phase-to-phase fault. Equation (20) shows the current in the loop and (21) shows the measured voltage at the relay.

$$I = \frac{E}{Z_S + Z} \tag{20}$$

$$V = \frac{E \cdot Z}{Z_S + Z} \tag{21}$$

These are then inserted into (2) and (3), multiplied by the common term (22), which yields (23) and (24).

$$\frac{Z_S + Z}{E} \tag{22}$$

$$S_1 = Z_R - Z \tag{23}$$

$$S_2 = Z \tag{24}$$

To apply pre-fault positive-sequence memory voltage to the comparator, E is substituted into (3), and following the same process yields (25).

$$S_2 = Z_S + Z \tag{25}$$

Equations (25) and (23) compare the total impedance to the fault, to the difference of the setting and the measured apparent impedance. The expansion comes from comparing the total impedance to the fault, rather than the measured apparent impedance. Essentially, there is a phase angle difference in the source (V1MEM) and the measured voltage (V), due to the voltage divider from the source to the relay.

C. *Mho Elements and Circles*

Phase comparators are difficult to visualize as a circle. The mho circles that come from these phase comparators are easier to visualize when analyzing magnitude comparators [13]. The magnitude and phase comparators produce the same output when the inputs are the same.

1) Magnitude Comparators

Magnitude comparators take the form of (26), with S_A and S_B defined in (27) and (28), respectively.

$$\frac{S_A}{S_B} = Me^{j\theta} \tag{26}$$

$$S_A = S_1 + S_2 \tag{27}$$

$$S_B = S_1 - S_2 \tag{28}$$

Equation (27) and (28) take the forms (29) and (30), respectively, when inputs (23) and (25) are used.

$$S_A = Z_R + Z_S \tag{29}$$

$$S_B = Z_R - 2 \cdot Z - Z_S \tag{30}$$

Using the magnitude comparator (26), and leaving off “M” [13], the equivalent takes the form of (31).

$$\frac{Z_R + Z_S}{Z_R - 2 \cdot Z - Z_S} = e^{j\theta} \tag{31}$$

Rearranging and solving for Z results in (32).

$$Z = \frac{-\left(\frac{Z_R + Z_S}{e^{j\theta}} + Z_S - Z_R\right)}{2} \tag{32}$$

Redistributing the negative sign and arranging with like terms results in (33).

$$Z = \frac{(Z_R + Z_S) \cdot e^{j\theta}}{2} + \frac{Z_R - Z_S}{2} \tag{33}$$

Equation (33) represents a circle with a radius of the reach plus the source impedance then divided by two, and an offset of the difference in reach and source impedances, divided by two. This is further visualized in fig. 20. If the measured impedance lies within the characteristic – within the directional elements operating range (forward) – the mho distance element will operate.

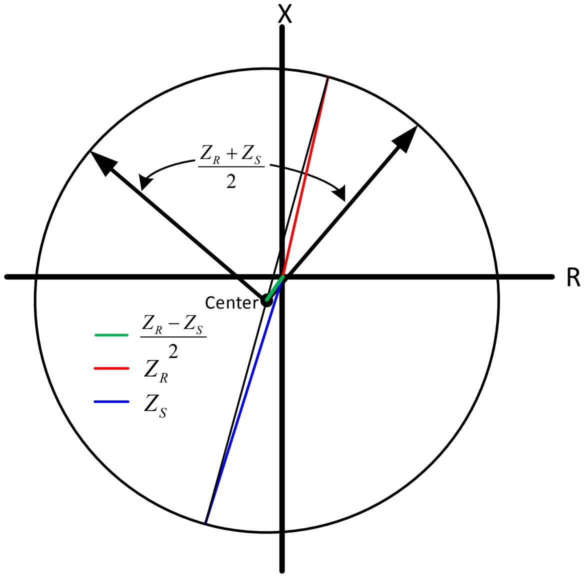

Fig. 20. Mho operating element region

From fig. 20, it is clear that the mho circles center point affects arc resistance coverage. In this specific example, the source is more resistive than the line, which is not likely in a looped transmission system. More than likely, the source impedance will be less resistive than the line, shifting the center point to the right, and thus, increasing the arc resistance coverage. Another way of looking at this is, the real component of (16) is more likely to be positive, or close to the negative X-axis when Z_S is mostly reactive (depending on the source impedances).

2) Phase Angle Comparators

The phase angle comparator takes the form (34), with a limited operating range theta [13]. This produces a half-circle operating range that is harder to visualize.

$$\frac{S_1}{S_2} = Me^{\pm j\theta} \tag{34}$$

Inputs (23) and (25) can be incorporated into (34), producing (35), and then a defined operating range of (36).

$$\frac{Z_R - Z}{Z_S + Z} = Me^{\pm j\theta} \tag{35}$$

$$(-90^\circ) < \angle S_1 - \angle S_2 < (90^\circ) \tag{36}$$

Phase angle comparators are dependent on variable phasors that are added or subtracted prior to comparing the angle difference. These are nearly impossible to simply analyze by hand due to the complex relationship in the phase angles. Since magnitude comparators yield the same result, they are recommended for furthering understanding.

D. Angle Changes with Arc Resistance

Phase angle comparators make visualizing the system impedance angle changes very difficult. Figure 21 appears quite busy at first glance. However, it conceptualizes the changes in angles as the arc resistance changes and as the source impedance increases, resulting in greater expansion. For example, R_F drives the measured Z and the comparator signal Z_R-Z away from each other for a self-polarized mho element. For a positive-sequence polarized mho element, ZP still moves away from the comparator signal Z_R-Z, but at a much slower rate, and even less the greater Z_S+Z is. If Z_S is large and highly reactive, Z_P starts off leading the line angle Z_L, then, depending on Z_S, it slowly becomes more lagging when compared to Z_L.

In other words, everything is fixed around the line angle from Z_L, and it is the system angle changes that determine whether the mho distance element will operate. The angle changes are determined by both the magnitude and angle of the vectors, which then affects the output of the phase angle comparator. So, even though the phase angle comparator focuses on the angles, the angles are dependent on both the magnitude and phase angles of the signals.

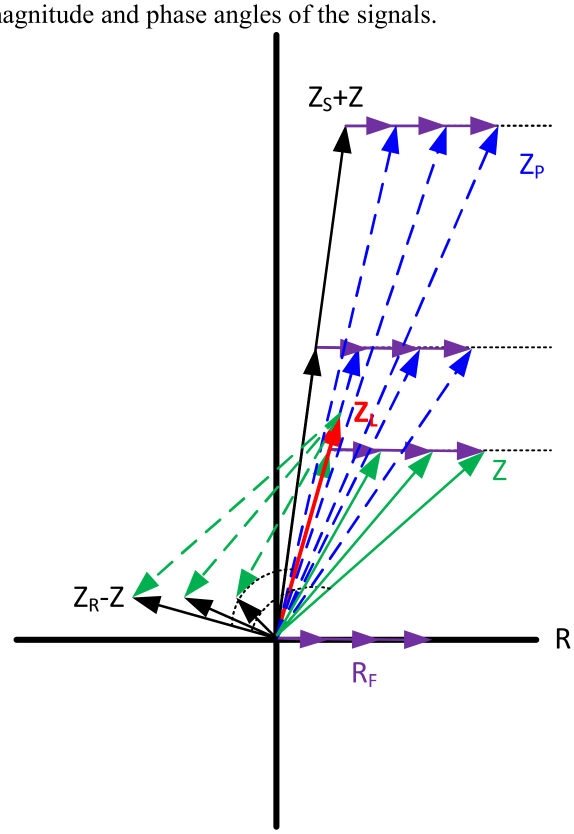

Fig. 21. Angle changes with arc resistance

VII. ACKNOWLEDGMENT

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