**Distance Protection, infeed effect and Resistance coverage-why Warrington cannot be ignored**

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*Several technical papers have been published in the past fifty years to describe the expansion of memory polarized Mho distance characteristics during faults. The higher arc coverage provided by expanded characteristics and its dependence on the source strength is well known. This paper presents calculations to determine the actual resistance coverage at any fault location on the lines protected by memory polarized distance elements. The paper also discusses the dependence of the resistance coverage on the source-to-line impedance ratio (SIR) and the type of fault.*

## 1. Introduction

Distance relay designs have used Mho distance elements to detect all types of faults on transmission lines. The use of phase comparators in the classical approach compared compensated voltage (V-IZ) with the polarizing voltage $V_P$, where V and I are fault voltages and currents, respectively. In the case of self-polarized Mho distance elements the fault voltage is used as the polarizing voltage in the phase comparator. Loss of discrimination for close-in faults where the fault voltage collapsed to zero led to the use of pre-fault voltage (memory polarization) as the polarizing voltage for all types of faults. It was observed that the use of memory voltage also resulted in the expansion of the relay impedance characteristics during faults and the expansion lasted as long as the memory voltage was present. Detailed explanation of arc resistance calculation and resistance coverage calculations are discussed in the following sections.

## 2. Arc Resistance

It has been shown that an arc is resistive in nature and arc resistance is dependent on the fault current. Warrington [1] provided an empirical formula to determine the arc resistance:

$$\mathrm{Rarc} = \frac{8750\,L}{\mathrm{I}^{1.4}} \tag{1}$$

Where L is the length of the arc in feet in still air and I is the fault current in amperes through the arc. Either phase-to-phase clearance or phase-to-tower distance is generally used as the length of the arc. Warrington also discussed the dynamic nature of the arc and the variation in the length of the arc due to cross winds and duration. Equation 1 modified with these inputs is

$$\mathrm{Rarc} = \frac{8750(L+3ut)}{\mathrm{I}^{1.4}} \tag{2}$$

Where, u is the crosswind velocity in miles per hour and t is the time in seconds.

An alternative formula provided in the Westinghouse Protection Book [2] was also based on empirical methods. It proposed a constant voltage drop per foot irrespective of the current through the arc:

$$\mathrm{Rarc} = \frac{440\ \mathrm{L}}{\mathrm{I}} \tag{3}$$

Both methods are extensively used in the power industry to determine the arc resistance.

The Westinghouse formula comes up with higher resistance for currents above 1763 A whereas Warrington's equation provides higher resistance for currents below this value.

As an example, for a 1000 amps fault current with arc length of 10ft, Warrington's formula would calculate the arc resistance as 5.52 ohms whereas Westinghouse formula would provide 4.4 ohms.

For a fault current of 10,000A, arc resistance is 0.22 ohms with Warrington's method and 0.44 with Westinghouse method.

The key point to be noted here is that the voltage drop across the arc either decreases with infeed or remains the same irrespective of the amount of infeed from the remote end.

## 3. Arc Resistance as seen by phase and ground distance Mho elements

Distance relay elements are designed to measure the positive sequence line impedance up to the fault location irrespective of the type of fault. To achieve this, three ground mho distance units and three phase mho distance units are required. Voltage and current inputs to these units, also referred to as elements, vary depending on their function as shown in the following table, Table-1:

| Mho Distance Element | Fault Voltage, V | Fault Current, I | Polarizing voltage (Pre-fault or positive sequence memory ) |
|---|---|---|---|
| A-G | $V_A$ | $I_A+K_0I_N$ | $V_{AP}$ |
| B-G | $V_B$ | $I_B+K_0I_N$ | $V_{BP}$ |
| C-G | $V_C$ | $I_C+K_0I_N$ | $V_{CP}$ |
| A-B | $V_{AB}$ | $I_A-I_B$ | $V_{ABP}$ |
| B-C | $V_{BC}$ | $I_B-I_C$ | $V_{BCP}$ |
| C-A | $V_{CA}$ | $I_C-I_A$ | $V_{CAP}$ |

Table 1: Voltage and Current inputs to Mho distance elements

### 3.1 Analysis of single line to ground arcing fault:

For A-G fault: $I_1 = I_2 = I_0 = I_A/3 = I_N/3$. For a transmission line $Z_{1L}= Z_{2L}$

Voltage at the relay location without any arc resistance, $V_A = I_1Z_{1L} + I_2Z_{2L}I+I_0Z_{0L}$ (4)

Voltage with arc resistance of R ohms, $V_A = I_1Z_{1L} + I_2Z_{2L}I+I_0Z_{0L} +I_AR$ (5)


Figure 1: Single line to ground fault with arc resistance R ohms

The ground distance element calculates the impedance up to the fault point by dividing the measured voltage on the faulted phase (A phase as an example) at the relay location by a current known as compensated current that includes faulted phase current and portion of neutral current ($I_A+K_0I_N$).

$$Z_{relay} = V_A/ [I_A+K_0I_N] \quad (6)$$

Rearranging the voltage equation (5), substituting $Z_{1L} =Z_{2L}$ and adding and subtracting $I_0Z_{1L}$

$$V_A = I_1Z_{1L} + I_2Z_{2L}I+I_0Z_{0L} +I_AR = [I_1Z_{1L} + I_2Z_{1L} + I_0Z_{1L} + I_0Z_{0L} - I_0Z_{1L} +I_AR \quad (7)$$

$$V_A = [I_1+I_2+I_0]\, Z_{1L} + I_N[Z_{0L}-Z_{1L}]/3 +I_AR \quad (8)$$

$V_A = I_A\, Z_{1L}+ I_NK_0\, Z_{1L} +I_AR$, where $K_0 = [Z_{0L}-Z_{1L}]/3\, Z_{1L}$, the zero sequence compensation factor

$$V_A = Z_{1L}\, [I_A+K_0I_N] \;\; + I_AR \quad (9)$$

For a radial line, $I_A = I_N$, the relay measures smaller arc resistance, $R/ (1+K_0)$.

$$\mathbf{Z_{relay} = V_A/ [I_A+K_0I_N] \;=\; Z_{1L} + R/[1+K_0]} \quad (10)$$

Actual fault resistance can include additional resistance such as tower footing resistance but the discussion here is focused only on the arc resistance. With shield wires grounded at every tower, the tower footing resistance effect is minimized. For transmission lines without shield wire, single line to ground fault currents will be low and use of directional ground overcurrent element is preferred to ground distance elements. The discussion is focused on the resistance reach where ground distance elements are used.

**If the Arc resistance coverage determined in a Mho characteristic is R ohms, the actual arc resistance coverage for single line ground fault is R $(1+K_0)$.**

3.2 Analysis of phase-to-phase arcing fault:

For an A-B arcing fault as shown in Figure 2

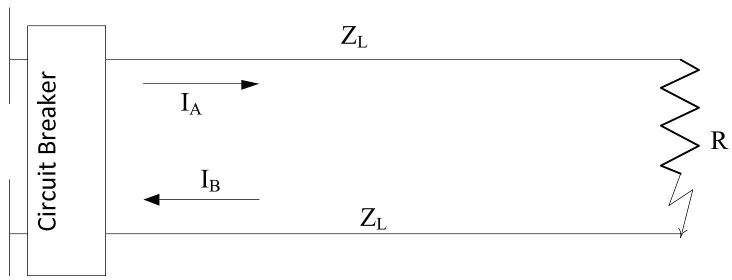


Figure 2: Phase to phase arcing fault with arc resistance R ohms

Voltage across faulted phase, $V_{AB} = (I_A-I_B)*Z_L + I_A*R$ (11)

Relay calculated impedance $ZR = (V_A-V_B)/ (I_A-I_B)$ (12)

$Z_R = Z_L+ I_A*R/ (I_A-I_B)$ (13)

$I_A= - I_B$,

$Z = Z_L+ R/2$ (14)

**If the resistance coverage determined in a Mho characteristic is R ohms, the actual arc resistance coverage for a phase-to-phase fault is 2*R**

Similarly, it can be shown that the actual arc resistance coverage during a three-phase fault has a factor of √3 greater resistance coverage than seen by the Mho distance element.

## 4. Dynamic expansion of Mho distance elements during faults

Figure 3 shows self-polarized and memory-polarized dynamic characteristics of Mho distance elements when the fault occurs. For a self-polarized Mho element there is no expansion of the characteristic whereas memory polarized Mho characteristic expands during the fault condition.

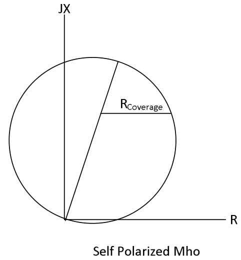


Figure 3a: Self-polarized Mho characteristic

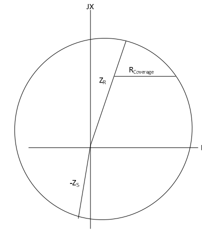


Figure 3b: Memory-polarized Mho characteristic

The degree of expansion is dependent on source impedance, and the expansion is greater for a weaker source or with higher source impedance. Both cross-polarized and memory-polarized produced the same effect but we will limit our discussions to only memory-polarization in this paper.

With memory voltage, the Mho characteristic expands with the diameter as ($Z_S$+$Z_R$), where $Z_S$ is the equivalent source impedance behind the relay and $Z_R$ is the relay reach. Polarized Mho relay characteristic expansion has been explained in detail in papers [3], [4] and [5].

$Z_S$ value depends on the type of the fault. For a three fault or a phase-to-phase fault, the source impedance value will be equal to the positive sequence source impedance, $Z_{1S}$ (same as the negative sequence impedance, $Z_{2S}$). For a single line-to-ground fault the equivalent source impedance would be [$Z_{1S}$+$Z_{2S}$+$Z_{0S}$]/3, where $Z_{0S}$ is the zero sequence source impedance.

## 5. Effect of Source to line impedance ratio on the resistance coverage

With finite source impedance, the diameter of the Mho distance element will be ($Z_S$+$Z_R$) with an offset Mho characteristic for forward faults as shown in Figure 4. $R_K$ will yield the maximum arc resistance coverage at $KZ_R$. To understand the effect of source impedance on the resistance coverage, system resistances are ignored initially. The system resistance's impact upon source impedance will be discussed in the next section.

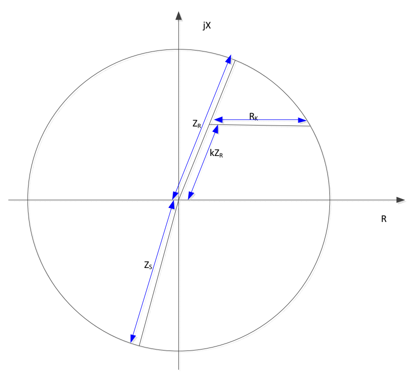


Figure 4: Expanded Mho characteristic showing arc resistance $R_K$ at a point $KZ_R$ on the line from relay location

Neglecting system resistances and assuming a system with system angle at 90 degrees, the determination of the resistance coverage can be reduced to determining the side of a triangle as shown in figure 5. It is also assumed that the relay reach is same as the line length.

$OA = (Z_S + Z_R)/2 - (1-K) Z_R = 0.5(Z_S - Z_R) + KZ_R$ ; $OB = (Z_S + Z_R)/2$

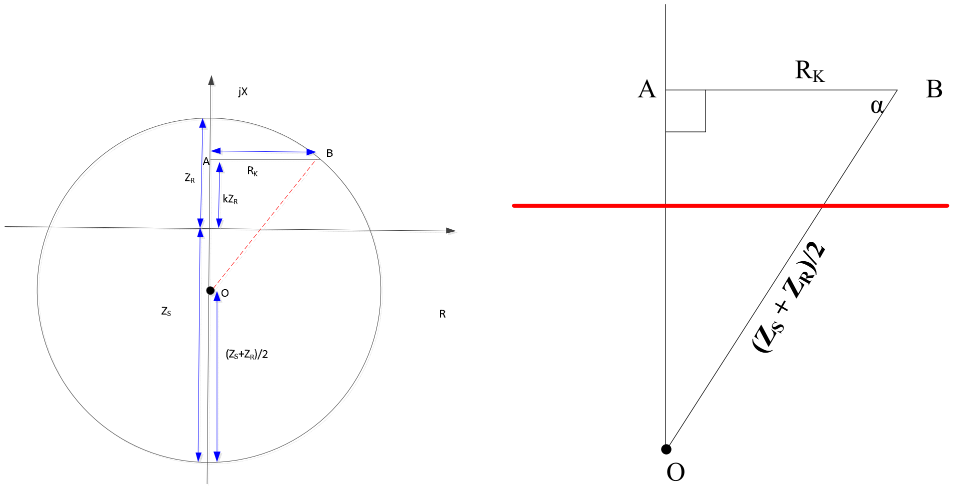


Figure 5: Expanded Mho characteristics assuming purely reactive impedances

$R_K = \sqrt{[((Z_S + Z_R)/2)^2 - (0.5(Z_S - Z_R) + KZ_R)^2]}$;
Simplifying the terms, $R_K = \sqrt{[(Z_S + KZ_R)(1-K)Z_R]}$
Replacing the source impedance with SIR*$Z_R$,

$$R_K = Z_R * \sqrt{[(1-K)(SIR+K)]} \quad (15)$$

For a close-in fault, K=0 and the resistance coverage $R_{close\text{-}in} = Z_R\sqrt{SIR}$ (16)

For an SIR of 5, the arc resistance coverage for a close in fault is 2.33*line impedance.

## 6. Effect of considering the resistance of the line and the source

In order to simplify the equation, the system is still considered homogenous (both source and line angles are the same). On examination of the characteristics, higher source angle would tilt the characteristics towards the resistive axis providing higher resistance coverage. The homogenous system assumption provides a more conservative value than the actual one at lower voltages.

The calculation can be reduced to a trigonometric problem to determine length of one side knowing two sides and one angle in a triangle and can be shown as in Figure 6.

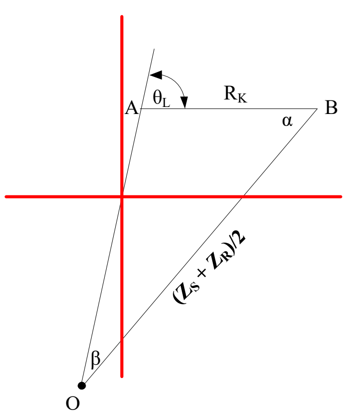


Figure 6: Determination of $R_K$ in the Triangle OAB.

OB = Radius of the circle, $(Z_S+Z_R)/2$;
AB= resistance coverage, $R_K$, to be determined
OA = $(Z_S + Z_R)/2 - (1-K)Z_R = 0.5(Z_S - Z_R) + KZ_R$
Angle OAB = $(180-\theta_L)$ where $\theta_L$ is the line angle in degrees.

Using the law of sines:
$(OA/\sin\alpha) = (OB/\sin OAB) = R_K/\sin\beta$
Angle α is determined from
$\sin\alpha = (OA/OB)\sin(OAB) = (OA/OB)*\sin(180-\theta_L)$

$$= (OA/OB)*\sin\theta_L = \frac{0.5(Z_S - Z_R) + KZ_R}{0.5(Z_S + Z_R)}\sin\theta_L$$

$$\text{Angle } \alpha = \sin^{-1}[\frac{0.5(Z_S - Z_R) + KZ_R}{0.5(Z_S + Z_R)} \sin\theta_L] \quad (15)$$

$$\text{Angle } \beta = 180 - [(180 - \theta_L) + \alpha] = (\theta_L - \alpha) \quad (16)$$

$$R_K = \frac{0.5(Z_S + Z_R)}{\text{Sin}\theta_L} \sin(\theta_L - \alpha) = \frac{0.5(Z_S + Z_R)}{\sin\theta_L} \sin[\theta_L - \sin^{-1}\left(\frac{0.5(Z_S - Z_R) + KZ_R}{0.5(Z_S + Z_R)} \sin\theta_L\right)] \quad (17)$$

Equation 17 calculates the actual resistance coverage based on the source impedance, fault location and the relay reach. In order to see the effect of source to line impedance ratio on the resistance coverage, let us assume the reach of the relay to be equal to line impedance, $Z_L$.
The resistance coverage at location K is given by:

$$R_K = [\frac{0.5(Z_S + Z_L)}{\text{Sin}\theta_L}]\text{Sin}(\theta_L - \alpha) = [\frac{0.5(Z_S + Z_L)}{\text{Sin}\theta_L}]\text{Sin}[\theta_L - \text{Sin}^{-1}\left(\frac{0.5(Z_S - Z_L) + KZ_L}{0.5(Z_S + Z_L)} \sin\theta_L\right)] \quad (18)$$

Replacing the source impedance, $Z_S$ with $SIR*Z_L$,

$$R_K = [\frac{0.5(SIR + 1)Z_L}{\sin\theta_L}]\sin[\theta_L - \sin^{-1}\left(\frac{0.5(SIR - 1)Z_L + KZ_L}{0.5(SIR + 1)Z_L} \sin\theta_L\right)] \quad (19)$$

The resistance coverage for a close-in fault with SIR = 0 is:

$$R_{Close\text{-}in} = \frac{0.5Z_L}{\sin\theta_L} \sin[\theta_L - \sin^{-1}\left(\frac{0.5(-1)Z_L}{0.5(1)Z_L} \sin\theta_L\right)] = \frac{0.5Z_L}{\sin\theta_L} \sin[\theta_L - \sin^{-1}(\sin(-\theta_L))$$

$$R_{Close\text{-}in\_SIR0} = \frac{0.5Z_L \sin(2\theta_L)}{\text{Sin}\theta_L} = Z_L\cos\theta_L \quad (20)$$

## 7. Conclusion:

Use of memory-polarized Mho distance elements resulted in the expansion of the characteristics during faults. This resulted in providing higher resistance coverage for faults on the transmission lines. The fault resistance coverage is dependent on SIR. The fault resistance seen by the Mho distance elements was smaller than the actual values for phase-to-ground and phase-to-phase faults and the degree of reduction depends on the type of fault.

## 8. References


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